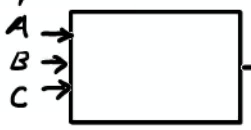
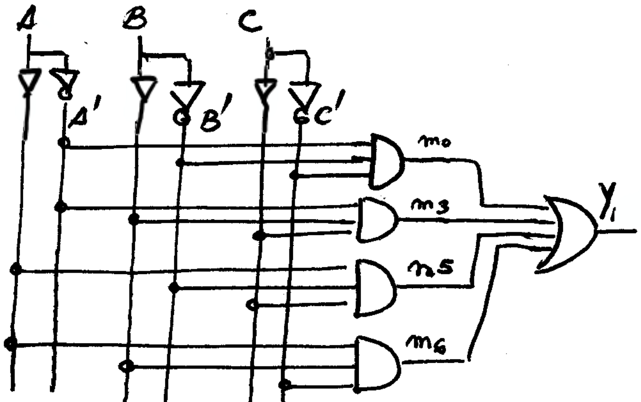
Designing logic circuits using only-NAND or only-NOR

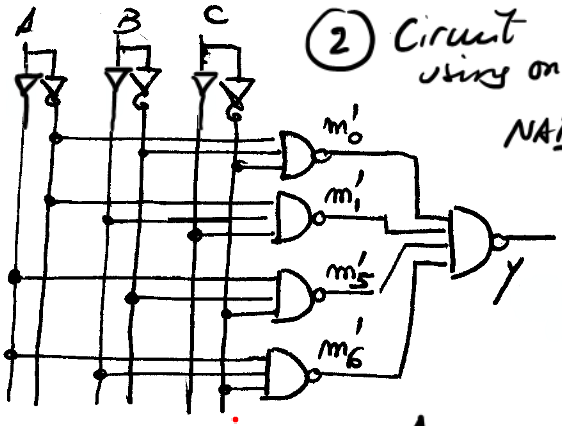
This is a simple example circuit  $\rightarrow Y = f(A,B,C)$

Truth table: $Y = \sum_3 m(0,3,5,6) = m_0 + m_3 + m_5 + m_6$
$= A'B'\cdot C' + A'BC + AB'C + ABC'$

(1) Canonical circuit



(2) Circuit using only NAND



using this transformation → $Y = (m_0 + m_3 + m_5 \cdot m_6)'' = (m_0' \cdot m_3' \cdot m_5' \cdot m_6')'$
only → NAND

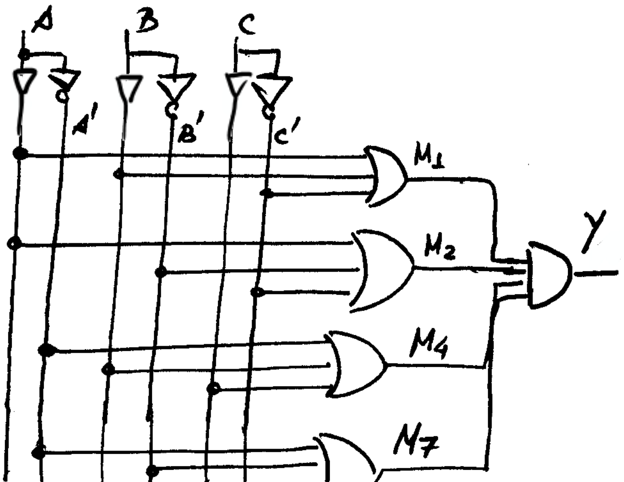
alternatively: (3) $Y = \prod_3 M(1,2,4,7) = M_1 \cdot M_2 \cdot M_4 \cdot M_7$ (OR, AND, NOT)

and with this transformation: $Y = (M_1 \cdot M_2 \cdot M_4 \cdot M_7)'' = (M_1' + M_2' + M_4' + M_7')'$ (NOR)

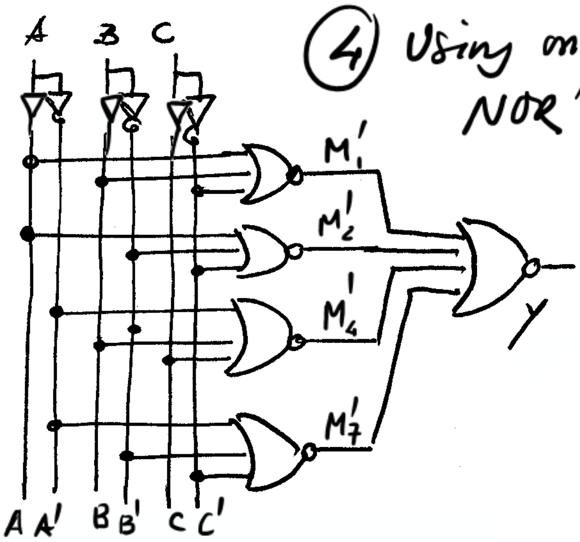
we have a circuit (4)

$(A+B+C')'$ $(A+B'+C)'$ $(A'+B+C)'$ $(A'+B'+C')'$

(3) canonical circuit using maxterms



(4) Using only NOR



These circuits have all of them 3 levels of gates

literals ⟷ functions ⟷ sum/product
(Buffers, inverters) (product/sum maxterms/minterms)

Thus, in principle, any combinational circuit can be implemented using only 3 levels of gates additionaly to the 3 levels of gats circuits, it is also possible to use only NAND or only NOR from any expression.

For instance: from circuit 3 (canonical based on maxterms)

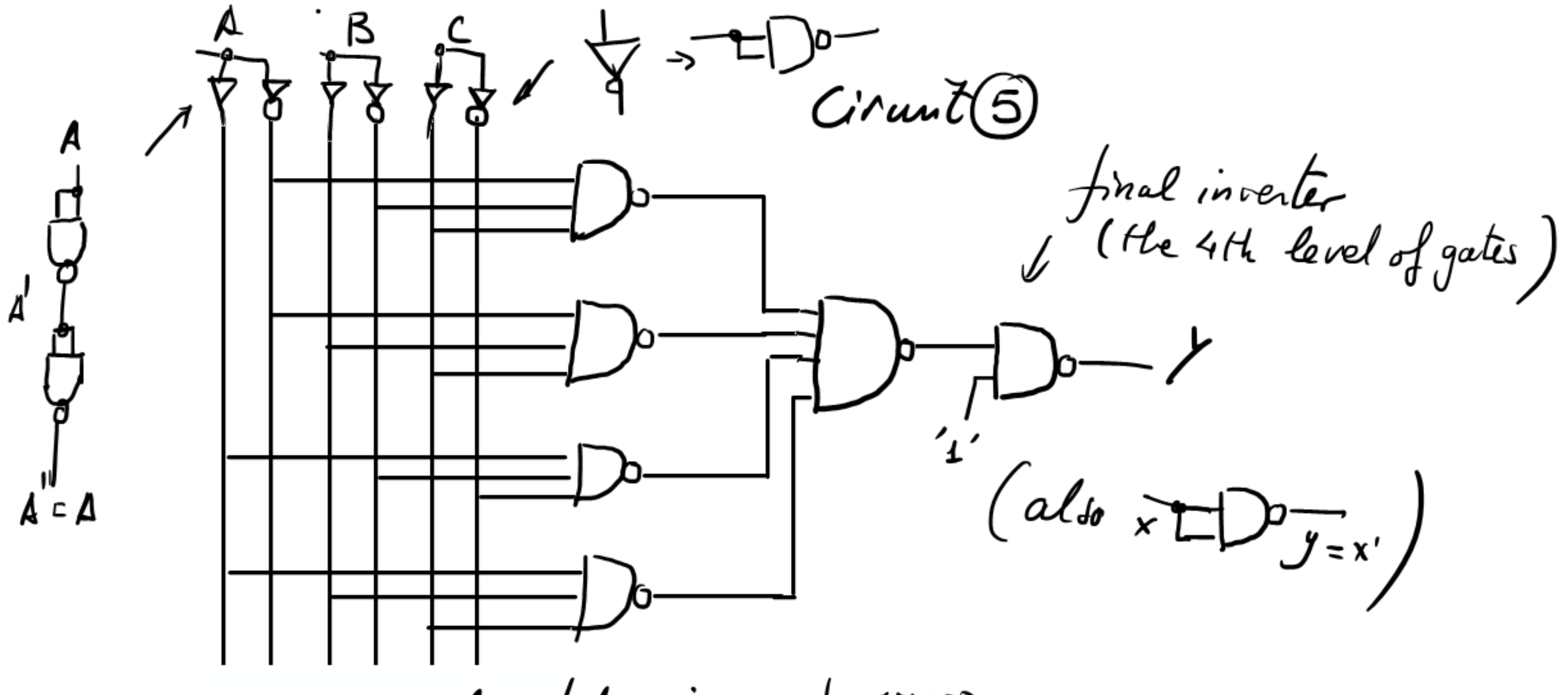
$$Y = \prod_3 M(1,2,4,7) = M_1 \cdot M_2 \cdot M_4 \cdot M_7$$

we can implement the circuit using only NAND → 5
adding another level of gates (a final inverter)

$$Y = (M_1'' \cdot M_2'' \cdot M_4'' \cdot M_7'')'' = \big((A+B+C')'' \cdot (A+B'+C)'' (A'+B+C)'' \cdot (A'+B'+C')'' \big)''$$

$$Y = \big((A' \cdot B' \cdot C'')' \cdot (A' \cdot B'' \cdot C')' \cdot (A'' \cdot B' \cdot C')' \cdot (A'' \cdot B'' \cdot C'')' \big)''$$

$$= \big(\big((A' \cdot B' \cdot C)' \cdot (A' \cdot B \cdot C)' \cdot (A \cdot B' \cdot C')' \cdot (A \cdot B \cdot C)' \big)' \big)'$$



All the circuit implemented using only NAND

In the same way we can design a complete circuit which contains only NOR.

For instance, from (1) $\quad y = \sum_3 m(0,3,5,6) = (m_0'' + m_3'' + m_5'' + m_6'')''$

$$y = \left((A' \cdot B' \cdot C')'' + (A' \cdot B \cdot C)'' + (A \cdot B' \cdot C)'' + (A \cdot B \cdot C')'' \right)''$$

$$y = \left(\left((A + B + C)' + (A + B' + C')' + (A' + B + C')' + (A' + B' + C)' \right)' \right)'$$

NOR $\qquad\qquad$ NOR $\qquad$ final inversion

Circuit (6)